Lecture 8

Today's agenda

- Product Differentiation: An Overview
- Hotelling's Linear City
- Salop's Circular City
- Discussion: Maximal or Minimal Differentiation?
- Problems

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Product Differentiation: An Overview I

- In most of the course so far (with some exceptions): models where the goods that firms produce are identical.
- In this lecture:
  - The products that firms produce are **differentiated**: they are not identical but still similar.
- Example:
  - Toothpaste and shaving cream are *different*, but two brands of toothpaste are *differentiated*.
- In terms of cross price elasticities:
  - Between toothpaste and shaving cream: low (or even zero).
  - Between two brands of toothpaste: significant.

Product Differentiation: An Overview II

Questions related to product differentiation that we want to understand

- Does product differentiation create market power and, if so, how much?
- How much will firms differentiate their products?
- What are the welfare implications of product differentiation? Does the market overprovide or underprovide variety?

Our main goals today

- Understand some economic forces that make firms want to move close to or far away from each other.
- Practice our analytical abilities.

Hotelling's Linear City I

Assumptions

- Each brand of a good is defined by its "location."
  - This could be a firm's location in physical or geogr. space.
  - Also: could be some other attributes of the good.
- Consumer preferences distributed in the same product space. A consumer's utility if not purchasing any brand is zero. A consumer's utility if purchasing brand i:

$$U(\theta^*, \theta_i) = \overline{s} - T(|\theta^* - \theta_i|) - p_i, \quad \text{where:}$$

  - θ_i is the location of the brand the consumer consumes;
  - θ^* is the consumer's ideal brand, or her location;
  - $T(\cdot)$ is an increasing function (measuring transportation costs or "mismatch costs");
  - p_i is the price the consumer must pay to get one unit of the brand;

Hotelling's Linear City II

- $\bar{s}$ is the utility that the consumer would get from consuming brand i for free and without having to travel.
  - Implicit assumption: if price and travel cost are the same across brands, the consumer is indifferent between the brands.

- Consumers have different preferences (or locations).
  - Assumptions:
    - Continuum of consumers (infinitely many, each infinitely small);
    - θ^* is a number between zero and one;
    - the distribution of θ^*s is **uniform** on that interval (i.e., consumers are spread out evenly).
    - Because there are positive transportation costs, each consumer chooses the firm that is nearest the own location.
    - Two firms that have chosen the same location split their joint market evenly.
    - The transportation cost function is quadratic:

$$T\left(|\theta^* - \theta_i|\right) = t\left(\theta^* - \theta_i\right)^2.$$

Hotelling's Linear City III

- Firms:
  - Two firms, 1 and 2 (entry of more firms not possible).
  - Firm 1's location: $\theta_1 = a$.
  - Firm 2's location: $\theta_2 = 1 - b$.$\rightarrow$ Assumption: $a < (1 - b)$
  - Constant marginal cost, c.
  - The firms want to maximize their profits or, equivalently, the number of consumers that buy their product.

- Timing of the game:
  1. The firms simultaneously choose their locations a and $1 - b$.
  2. The firms observe the rival's location. Then they simultaneously choose prices, p_1 and p_2.
  3. Each consumer chooses from which firm to buy.

- Equilibrium concept:
  - Subgame perfect Nash equilibrium (SPNE).
    - Since we have a finite horizon, we can solve for the SPNE by backward induction.

Hotelling's Linear City IV

Analysis

1. Solve the typical consumer's problem for given prices.
2. This gives us the demand function for each firm.
3. Formulate expressions for the firms' profit functions and solve for a Nash equilibrium (in prices) between the two firms.
4. Plug the derived equilibrium prices into each firm's profit function. $\rightarrow$ Profits as a function of a and b.
5. Solve for a Nash equilibrium (in locations) between the two firms.

Hotelling's Linear City V

The typical consumer's problem and the demand functions

The methodology

- Identify the consumer who, given some prices and locations, is indifferent between the two firms.
  - All consumers left of this consumer will prefer firm 1;
  - and everyone left of the indifferent consumer will prefer firm 2.
- This gives us the demand functions.

Assumption

- $\bar{s}$ is so large that all consumers always buy (the market is *covered*).
  - More generally, it could be that a consumer preferred not to buy at all (and get utility zero).

Hotelling's Linear City VI

Finding the indifferent consumer

- A consumer with ideal location θ^* gets the following utility if buying from Firm 1:

$$U(\theta^*, \theta_1) = \overline{s} - t(\theta^* - a)^2 - p_1.$$

- A consumer with ideal location θ^* gets the following utility if buying from Firm 2:

$$U(\theta^*, \theta_2) = \overline{s} - t(\theta^* - 1 + b)^2 - p_2.$$

- A consumer with ideal location $\overline{\theta}$ is indifferent between the two firms, where $\overline{\theta}$ satisfies

$$\overline{s} - t\left(\overline{\theta} - a\right)^2 - p_1 = \overline{s} - t\left(\overline{\theta} - 1 + b\right)^2 - p_2.$$

Hotelling's Linear City VII

- Solving for $\overline{\theta}$, we have

$$\overline{\theta} = \frac{p_2 - p_1}{2t(1 - a - b)} + \frac{a + 1 - b}{2}.$$

Finding the demand functions

- Firm 1 gets all the consumers left of $\overline{\theta}$. Therefore, its sold quantity is

$$D_1(p_1, p_2) = \overline{\theta} = \frac{p_2 - p_1}{2t(1 - a - b)} + \frac{a + 1 - b}{2}.$$

- Similarly, Firm 2's sales are

$$D_2(p_1, p_2) = 1 - \overline{\theta} = \frac{p_1 - p_2}{2t(1 - a - b)} + \frac{b + 1 - a}{2}.$$

Hotelling's Linear City VIII

Profit functions and second stage Nash equilibrium

- Firm 1's profits are

$$\begin{aligned}\Pi_1(p_1, p_2) &= (p_1 - c) D_1(p_1, p_2) \\ &= (p_1 - c)\left[\frac{p_2 - p_1}{2t(1-a-b)} + \frac{a+1-b}{2}\right].\end{aligned}$$

- Similarly, Firm 2's profits are

$$\begin{aligned}\Pi_2(p_1, p_2) &= (p_2 - c) D_2(p_1, p_2) \\ &= (p_2 - c)\left[\frac{p_1 - p_2}{2t(1-a-b)} + \frac{b+1-a}{2}\right].\end{aligned}$$

Hotelling's Linear City IX

- Let's look for a Nash equilibrium of the game between the two firms!
  - Firm 1's FOC:

$$\left[\frac{p_2 - p_1}{2t(1-a-b)} + \frac{a+1-b}{2}\right] - \frac{p_1 - c}{2t(1-a-b)} = 0.$$

  - Firm 2's FOC:

$$\left[\frac{p_1 - p_2}{2t(1-a-b)} + \frac{b+1-a}{2}\right] - \frac{p_2 - c}{2t(1-a-b)} = 0.$$

  - Solving yields

$$p_1^*(a, b) = c + t(1-a-b)\left(1 + \frac{a-b}{3}\right), \tag{1}$$

$$p_2^*(a, b) = c + t(1-a-b)\left(1 + \frac{b-a}{3}\right). \tag{2}$$

Hotelling's Linear City X

First stage Nash equilibrium

- Firm 1's profit is

$$\Pi_1(a,b) = [p_1^*(a,b) - c]\, D_1[a, b, p_1^*(a,b), p_2^*(a,b)].$$

- Firm 2's profit is

$$\Pi_1(a,b) = [p_2^*(a,b) - c]\, D_2[a, b, p_1^*(a,b), p_2^*(a,b)].$$

- **Claim**. There is only one Nash equilibrium. In this equilibrium, the two firms locate at the extremes of the city ($a = 0$ and $b = 0$).
- *Proof*. It suffices to show that $\Pi_1(a,b)$ is decreasing in a.
  - Note that the effect on $\Pi_1(a,b)$ of a change in a that goes through $p_1^*(a,b)$ is zero (due to the envelope theorem).
  - Therefore, $\partial\Pi_1(a,b)/\partial a$ is equal to the sum of the following two terms:

Hotelling's Linear City XI

1. The direct effect on demand of a change in a:

$$\begin{aligned} & \frac{\partial}{\partial a}\left[\frac{p_2 - p_1}{2t(1-a-b)} + \frac{a+1-b}{2}\right] \\ = & \frac{p_2 - p_1}{2t(1-a-b)^2} + \frac{1}{2} \\ = & \frac{3-5a-b}{6(1-a-b)}, \end{aligned}$$

where the last equality follows from substituting (**??**) and (**??**) and simplifying (verify this yourself!).

Hotelling's Linear City XII

2. The effect on demand of a change in a that goes through the rival's price, $p_2^*(a,b)$:

$$\begin{aligned}
& \frac{\partial D_1(p_1,p_2)}{\partial p_2}\frac{\partial p_2^*(a,b)}{\partial a} \\
= & \frac{1}{2t(1-a-b)} \times \\
& t\left[-\left(1+\frac{b-a}{3}\right)-\frac{1}{3}(1-a-b)\right] \\
= & \frac{1}{2t(1-a-b)} t\left[-\frac{4}{3}+\frac{2a}{3}\right] \\
= & -\frac{4-2a}{6(1-a-b)}.
\end{aligned}$$

Hotelling's Linear City XIII

- Hence,

$$\begin{aligned}
& \frac{\partial \Pi_1(a,b)}{\partial a} \\
= & \left[p_1^*(a,b)-c\right]\left[\frac{3-5a-b}{6(1-a-b)}-\frac{4-2a}{6(1-a-b)}\right] \\
= & -\left[p_1^*(a,b)-c\right]\left[\frac{1+3a+b}{6(1-a-b)}\right] < 0
\end{aligned}$$

- We can interpret the effects above:
  - Effect 1 (the market-share effect): For a not too large, increasing a makes profits larger as you steal market shares from the rival.
  - Effect 2 (strategic eff.): As you get closer to your rival, price competition intensifies and this has a negative impact on your profit.

Hotelling's Linear City XIV

- Conclusion:
  - The model gives rise to max. differentiation.
- A benevolent social planner who wanted to minimize transportation costs (recall that consumption is fixed) would choose the location $\frac{1}{4}$ and $\frac{3}{4}$.

Salop's Circular City I

Assumptions

- A continuum of consumers in a market are uniformly distributed on a circle.
- A large number of firms simultaneously choose whether or not to enter the market.
- The firms that choose to enter are (exogenously and automatically) located equidistant from one another on the circle.
- When the firms have entered they simultaneously choose their prices.
- Finally, the consumers (knowing the locations and prices of all firms) choose from which firm to buy one unit of the good.
  - The consumers either buy one good or no good at all, and the parameters of the model are such that, in equilibrium, all consumers indeed buy.

Salop's Circular City II

- Let n denote the number of entering firms.
- Let t denote the consumers' unit transport cost.
- The firms incur a fixed cost f if and only if they enter.
- If they do enter, they incur a constant marginal cost of c for each unit they sell.

Analysis

- Look for an equilibrium in which n firms enter and all charge the price p.
- Consider the decision of one of the firms. Its price is denoted p_i.
- It faces competition only from its two closest neighbors (who both in equilibrium charges the price p).
  - Since there are n equidistant firms, the distance between any two of them is $1/n$.

Salop's Circular City III

- A consumer located between our firm and one of its neighbors, say at x, is indifferent between the firms if

$$\underbrace{p_i + tx}_{\text{Cost of buying from our firm.}} = \underbrace{p + t\left(\frac{1}{n} - x\right)}_{\text{Cost of buying from rival firm.}}$$

- Solving for x:

$$x = \frac{p + t/n - p_i}{2t}.$$

- Since our firm has two neighbors and on each side gets the consumers between itself and the point x (and the consumers are uniformly distributed on that interval), our firm's demand is

$$D(p_i, p) = 2x = \frac{p + t/n - p_i}{t}.$$

Salop's Circular City IV

- Therefore, our firm solves

$$\max_{p_i} \left[(p_i - c) \left(\frac{p + t/n - p_i}{t} \right) - f \right]$$

- Taking the FOC:

$$\left(\frac{p + t/n - p_i}{t} \right) - \frac{1}{t} (p_i - c) = 0.$$

- Setting $p_i = p$ and solving yield

$$p^* (n) = c + \frac{t}{n}.$$

- Plugging this back into a typical firm's profit function:

$$\begin{aligned} \Pi_i &= \left[(p^* - c) \left(\frac{p^* + t/n - p^*}{t} \right) - f \right] \\ &= \frac{t}{n^2} - f. \end{aligned}$$

Salop's Circular City V

- Firms will enter as long as they can get a positive profit. Therefore, the equilibrium number of firms:

$$n^* = \sqrt{\frac{t}{f}}$$

(or, since n^* must be a whole number, the integer closest to or not exceeding n^*).

- And the market price with the equilibrium number of firms:

$$p^* (n^*) = c + \frac{t}{n^*} = c + \sqrt{tf}.$$

Conclusions

- Number of firms n^* increasing in transportation cost t and decreasing in fixed costs f.
- The profit margin $(p^* (n^*) - c)$ increasing in transportation cost t and in fixed costs f.

Salop's Circular City VI

- Price exceeds marginal cost ($p^*(n^*) > c$), but still firms don't make profits.
- We approach marginal cost pricing if either the transportation cost t or the fixed cost f goes to zero.
  - Compare the Bertrand model.

Socially Optimal Number of Firms

- Suppose a social planner who cares about the sum of the fixed costs and the consumers' transportation costs could choose n.
  - Would he choose an n that is higher or lower than n^*?

Salop's Circular City VII

- The planner's problem: minimize (w.r.t. n)

$$\begin{aligned} L(n) &= nf + t\left(2n\int_0^{1/2n} x dx\right) \\ &= nf + 2tn\left[\frac{1}{2}x^2\right]_0^{1/2n} \\ &= nf + tn\left[x^2\right]_0^{1/2n} = nf + \frac{tn}{4n^2} \\ &= nf + \frac{t}{4n}. \end{aligned}$$

Salop's Circular City VIII

- The FOC:

$$f - \frac{t}{4n^2} = 0$$

  or

$$n = \frac{1}{2}\sqrt{\frac{t}{f}},$$

  which is exactly one-half of n^*.
- Conclusion:
  - The market gives rise to too much entry.
  - Intuition: A "business-stealing effect" (or "trade-diversion effect" in Tirole): the firms have a private incentive to enter and take sales from the other firms — but this does not add to aggregate welfare as this just amounts to a transfer of profits from one firm to another.

Discussion: Maximal or Minimal Differentiation? I

- In the Hotelling model we identified one effect that works in the direction of maximal differentiation: firms want to differentiate in order to soften price competition. There are, however, forces that work in the opposite direction:
  - *Be where the demand is.*
    - We had this effect in the Hotelling model, even though the other effect was stronger.
  - *Positive externalities between firms.*
    - Examples: fishermen converge to the same harbor, firms locate near a common source of raw material, many similar shops on the same street lower the consumers search costs and increases aggregate demand.
  - Absence of price competition.
    - Because of regulatory rules, the price in a market may be fixed. In such cases, the incentive to differentiate should decrease.
    - Hotelling's "principle of *minimal differentiation*" (cf. the political economy literature on elections).

Problems I

Problem 1:
Prove that in "Hotelling's Linear-City Model with Exogenous Prices and Two Firms" there is a unique equilibrium in which both firms locate exactly in the middle. Why is this equilibrium suboptimal from a social welfare point of view (give the social welfare criterion used in the lecture notes)?

Problem 2:
Consider the following Hotelling's linear city model with endogenous prices and exogenous and symmetric locations. Suppose, that there is only one firm, and that this monopolist is (exogenously) located at the left end point of the interval (i.e., θ_1=0). Also, allow for the possibility that some consumers may prefer not to buy at all. Solve for the optimal monopoly price in this model.

Problems II

- Two firms
  - Locations are fixed and give by $\theta_1 = x$ and $\theta_2 = 1 - x$, so the firms choose only their prices (assume $x < \frac{1}{2}$).
  - No production costs.
  - Everything else exactly as before.
- Consumers
  - The transportation cost function is quadratic:

$$T(|\theta^* - \theta_i|) = k(\theta^* - \theta_i)^2$$

  - Everything else exactly as before
- Timing of the game:
  1. The firms simultaneously choose a price
  2. Each consumer chooses from which firm to buy.
- Equilibrium concept
  - Subgame perfect Nash equilibrium
  - $\rightarrow$ Solve by backward induction